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Use of Assembled Functions Theory for Computing Dynamic Response of a Tunnel

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Extended Abstract

An analytical solution for the evaluation of dynamic response of a tunnel in infinite isotropic elastic porous media is presented. Tunnel is considered as a circular cavity. Two groups of complex functions for solid skeleton and pore fluid in a two-dimensional (2D) complex plane are introduced in order to solve the Biot equations. Stress, displacement and pore pressure fields induced by incident and scattered waves in the medium and especially in the vicinity of the cavity are evaluated in this complex plane. The validation of the proposed solution is shown by various numerical examples. A parametric study including the effects of fluid compressibility changes, shear modulus and permeability variations, several wave numbers and wave types (fast, slow and shear waves) is performed.

1. Introduction

Scattering of waves by a cavity in an isotropic elastic porous medium is of significant interest in geomechanical and engineering problems such as underground structure design (tunnels and piles). For a linear elastic solid (non-porous), exact solutions for simple geometry are well known (Pao et al., 1973; Erigen and Suhubi, 1964). Wave propagation and scattering in porous media can be described by the theory given by Biot (1956). Gatmiri (1992) and Jeng (1997), among others, have proposed the analytical solutions for these equations in special cases (harmonic wave loading in quasi static condition). Gatmiri (1990) has proposed a finite element solution for the same case in a general geometry and loading. For the dynamic case, Degrande and De Roeck (1993), have studied the problem by using the finite element method and Keynia (1992), and Dominguez (1992) by using the boundary element method. While the wave scattering by a cavity in porous media has been of great interest in different fields but not so many solutions are available. Ziemmerman and Stern (1993) extended the previous methods for the solid media to the porous media. Mei et al. (1984) have studied the scattering problem by using a boundary layer approximation method.

This paper presents an analytical solution for the scattering of harmonic waves in infinite isotropic porous medium based on complex function theory under assumption of plane strain. The complex functions in elasto-static problems were used by Muskhelishvili (1963). Nowinski (1982) has proposed a solution for static stress concentration around a hole subjected to uniaxial tension by the complex function approach. This method has been used for the evaluation of wave scattering by a cavity in infinite elastic solid by Liu et al. (1982).Degrande et al. (2017).have proposed a numerical model for ground-borne vibrations from underground railway traffic based on a periodic finite element–boundary element formulation. Alielahi et al. (2016) have also proposed a BEM investigation on the influence of

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underground cavities on the seismic response of canyons.

In this paper, an extension of complex functions application for the wave scattering is presented for the first time. In this approach, two groups of potential functions, (Φ^s, Ψ^s and Φ^f, Ψ^f) are introduced. The governing equations are transformed in the complex plane. By solving an eigenvalue problem, the relation between potential functions and is determined, while the relation between functions, Ψ^s, Ψ^f is found directly. In the complex plane, the solution of the resulting partial differential equations has been presented in series of the Hankel functions (complex sum of the Bessel functions) with unknown coefficients. It is obvious that the Hankel functions satisfy the radiation conditions in a dynamic problem in a simple and direct manner. By applying appropriate boundary conditions, a set of algebraic equations gives the unknown coefficients.

In order to validate the proposed solution, some comparisons are shown. Stress, pore pressure and displacement fields in vicinity of a tunnel in an infinite, isotropic and elastic porous medium subjected to P and SV harmonic waves are presented. The effects of the variation of important parameters such as the shear modulus, the permeability of the soil and the wave number are studied.

2. Governing Equations

For an isothermal fully saturated porous medium, two field variables related to the solid skeleton and to the fluid phase should be defined: the displacement of the solid skeleton, u_i and the average relative displacement of the fluid with respect to the solid skeleton w_i, which is measured in terms of volume of fluid per unit total cross section area of the bulk medium. Considering the displacement of the fluid as v_i, one can write, $w_i = n\ (v_i - u_i)$ where n is the porosity of the soil medium. The equilibrium equation and the modified Darcy equation of the saturated porous media can be written as:

$$\begin{aligned} &\sigma_{ij,j} + \rho b_i = \rho \ddot{u}_i + \rho_f \ddot{w}_i \\ &-p_{,i} + \rho_f b_i = \rho_f \ddot{u}_i + \frac{\rho_f}{n}\ddot{w}_i + \frac{1}{k}\dot{w}_i \end{aligned} \tag{1}$$

where σ_{ij} is total stress tensor component, ρb_i and $\rho_f b_i$ are the body forces and u_i, v_i are the displacement vector components of solid skeleton and pore fluid respectively. n is the porosity of the soil, P is the pore fluid pressure and k is the permeability of the soil. $\rho = (1-n)\rho_s + n\rho_f$ in which ρ, ρ_f and ρ_s are the mass density of mixture, pore fluid and solid skeleton respectively. The dots (.) denote the derivation respect to the time ($\frac{\partial}{\partial t}$). The poroelastic constitutive equations can be written in the plane strain condition can be written as:

$$\begin{aligned} &\sigma_{ij} = 2G\varepsilon_{ij} + \lambda e - \delta_{ij}\alpha p \\ &-p = \alpha Q e + Q\vartheta \end{aligned} \tag{2}$$

where ε_{ij} is the strain tensor component, λ and G are the Lamé elastic coefficients. e is the volumetric change in the solid skeleton and ϑ is the volume of pore fluid going out from a unit volume of bulk material. α is the Biot coefficient, Q is the Biot modulus and δ_{ij} is the Kronecker delta. From (1) and (2), the equilibrium equations in the absence of body forces in terms of displacement, are derived as:

$$\begin{aligned} &G\nabla^2 u_i + \left(\lambda + G + \alpha^2 Q\right)e_{,i} + \alpha Q\vartheta_{,i} = \rho\ddot{u}_i + \rho_f\ddot{w}_i \\ &\alpha Q e_{,i} + Q\vartheta_{,i} = \rho_f\ddot{u}_i + \frac{\rho_f}{n}\ddot{w}_i + \frac{1}{k}\dot{w}_i \end{aligned} \tag{3}$$

3. Potential Functions

In the two-dimensional (2D) case, the potential functions Φ (for P wave) and Ψ (for SV wave) are defined as follows (Φ^s, Ψ^s for solid skeleton and Φ^f, Ψ^f for pore fluid):

$$\begin{cases} u_x = \dfrac{\partial\Phi^s}{\partial x} + \dfrac{\partial\Psi^s}{\partial y} & w_x = \dfrac{\partial\Phi^f}{\partial x} + \dfrac{\partial\Psi^f}{\partial y} \\ u_y = \dfrac{\partial\Phi^s}{\partial y} - \dfrac{\partial\Psi^s}{\partial x} & w_y = \dfrac{\partial\Phi^f}{\partial y} - \dfrac{\partial\Psi^f}{\partial x} \end{cases} \tag{4}$$

where u_x, u_y and w_x, w_y are the in plane components of the vectors u_i and w_i respectively. Therefore, equation (3) in the harmonic case ($\Phi = \phi e^{-i\omega t}, \Psi = \psi e^{-i\omega t}$) is converted to the two groups of equations:

$$\begin{cases} (\lambda + 2G + \alpha^2 Q)\nabla^2\phi^s + \alpha Q\nabla^2\phi^f = -\rho\omega^2\phi^s - \rho_f\omega^2\phi^f \\ \alpha Q\nabla^2\phi^s + Q\nabla^2\phi^f = -\rho_f\omega^2\phi^s - \left(\frac{\rho_f}{n}\omega^2 + i\frac{\omega}{k}\right)\phi^f \end{cases} \quad (5)$$

$$\begin{cases} G\nabla^2\psi^s = -\rho\omega^2\psi^s - \rho_f\omega^2\psi^f \\ 0 = -\rho_f\omega^2\psi^s - \left(\frac{\rho_f}{n}\omega^2 + i\frac{\omega}{k}\right)\psi^f \end{cases} \quad (6)$$

where ∇^2 is the 2D Laplace operator, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and ω is the frequency of the waves.

4. Complex Functions

The complex variables are introduced as:

$$\zeta = x + iy \qquad \bar{\zeta} = x - iy \qquad \zeta = re^{i\theta} \quad (7)$$

Using relations:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \bar{\zeta}} \quad ; \quad \frac{\partial}{\partial y} = i\left(\frac{\partial}{\partial \zeta} - \frac{\partial}{\partial \bar{\zeta}}\right) \quad (8)$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \zeta^2} + 2\frac{\partial^2}{\partial \zeta \partial \bar{\zeta}} + \frac{\partial^2}{\partial \bar{\zeta}^2} \; ;$$
$$\frac{\partial^2}{\partial y^2} = -\left(\frac{\partial^2}{\partial \zeta^2} - 2\frac{\partial^2}{\partial \zeta \partial \bar{\zeta}} + \frac{\partial^2}{\partial \bar{\zeta}^2}\right) \quad (9)$$

The Laplace operator in the complex plane is obtained as:

$$\nabla^2 = 4\frac{\partial^2}{\partial \zeta \partial \bar{\zeta}} \quad (10)$$

Therefore, equations (5) and (6) are written as:

$$\begin{cases} 4(\lambda + 2G + \alpha^2 Q)\frac{\partial^2\phi^s}{\partial\zeta\partial\bar{\zeta}} + 4\alpha Q\frac{\partial^2\phi^f}{\partial\zeta\partial\bar{\zeta}} \\ + \rho\omega^2\phi^s + \rho_f\omega^2\phi^f = 0 \\ 4\alpha Q\frac{\partial^2\phi^s}{\partial\zeta\partial\bar{\zeta}} + 4Q\frac{\partial^2\phi^f}{\partial\zeta\partial\bar{\zeta}} + \rho_f\omega^2\phi^s \\ + \left(\frac{\rho_f}{n}\omega^2 + i\frac{\omega}{k}\right)\phi^f = 0 \end{cases} \quad (11)$$

$$\begin{cases} 4G\frac{\partial^2\phi^s}{\partial\zeta\partial\bar{\zeta}} + \rho\omega^2\psi^s + \rho_f\omega^2\psi^f = 0 \\ \rho_f\omega^2\psi^s + \left(\frac{\rho_f}{n}\omega^2 + i\frac{\omega}{k}\right)\psi^f = 0 \end{cases} \quad (12)$$

The following potential functions can be considered:

$$\begin{cases} \phi^s = \sum_{n=-\infty}^{+\infty} a_n^s H_n^{(1)}(\delta r)e^{in\theta} \\ \psi^s = \sum_{n=-\infty}^{+\infty} b_n^s H_n^{(1)}(\beta r)e^{in\theta} \end{cases}$$
$$\begin{cases} \phi^f = \sum_{n=-\infty}^{+\infty} a_n^f H_n^{(1)}(\delta r)e^{in\theta} \\ \psi^f = \sum_{n=-\infty}^{+\infty} b_n^f H_n^{(1)}(\beta r)e^{in\theta} \end{cases} \quad (13)$$

where $H_n^{(1)}(\chi r)$ is the Hankel function of first kind and order n:

$$H_n^{(1)}(\chi r) = J_n(\chi r) + iY_n(\chi r), \quad (14)$$

in which $J_n(\chi r)$ and $Y_n(\chi r)$ are the Bessel functions of first and second kind respectively and of order n.

$$J_n(\chi r) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\chi r}{2}\right)^{2k+n}}{k!\,\Gamma(n+k+1)} \quad (15)$$

$$Y_n(\chi r)=\frac{2}{\pi}J_n(\chi r)\left[Ln\left(\frac{\chi r}{2}\right)+\gamma\right] -\frac{1}{\pi}\sum_{k=0}^{n-1}\frac{(n-k-1)!}{k!}\left(\frac{\chi r}{2}\right)^{2k-n} -\frac{1}{\pi}\sum_{k=0}^{\infty}(-1)^k\frac{\left(\frac{\chi r}{2}\right)^{2k+n}}{k!(n+k)!}[\varphi(k)+\varphi(n+k)] \tag{16}$$

This type of the Hankel function satisfies the radiation boundary condition that states the waves, which travel to infinity, should not return (Sommerfield condition). $a_n^s, b_n^s, a_n^f, b_n^f$ are the unknown coefficients. Using the following relations:

$$\frac{\partial}{\partial\zeta}\left[H_n^{(1)}(\chi r)e^{in\theta}\right]=\frac{\chi}{2}H_{n-1}^{(1)}(\chi r)e^{i(n-1)\theta} \tag{17}$$

$$\frac{\partial}{\partial\bar{\zeta}}\left[H_n^{(1)}(\chi r)e^{in\theta}\right]=-\frac{\chi}{2}H_{n+1}^{(1)}(\chi r)e^{i(n+1)\theta} \tag{18}$$

Equation (11) can be converted to an eigenvalue problem as:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}\begin{Bmatrix}\phi^s \\ \phi^f\end{Bmatrix}=0\,, \tag{19}$$

in which the coefficients a_{ij} are:

$$\begin{cases} a_{11}=\left(\lambda+2G+\alpha^2Q\right)\delta^2-\rho\omega^2 \\ a_{12}=\alpha Q\delta^2-\rho_f\omega^2 \\ a_{21}=\alpha Q\delta^2-\rho_f\omega^2 \\ a_{22}=Q\delta^2-\left(\frac{\rho_f}{n}\omega^2+i\frac{\omega}{k}\right)\end{cases} \tag{20}$$

For this eigenvalue problem, determinant of the coefficients matrix should be equal to zero, therefore two quantities for δ are computed, δ_1 for P1 (fast dilatational wave) and δ_2 for P2 (slow dilatational wave). Knowing these parameters, the relation between potential functions of the solid skeleton and pore fluid for P wave is derived:

$$\phi_j^f=C_j\phi_j^s \qquad C_j=-\frac{\left(\lambda+2G+\alpha^2Q\right)\delta_j^2-\rho\omega^2}{\alpha Q\delta_j^2-\rho_f\omega^2} \qquad j=1,2 \tag{21}$$

From equation (12), the relation between the potential functions of the solid skeleton and pore fluid for SV wave can be derived directly:

$$\psi^f=D\psi^s \qquad D=-\frac{\rho_f\omega^2}{\left(\frac{\rho_f}{n}\omega^2+i\frac{\omega}{k}\right)} \qquad \beta=\left[\frac{\rho\omega^2+\rho_f\omega^2D}{G}\right]^{1/2} \tag{22}$$

Finally, the complete solution can be found:

$$\begin{cases}\phi^s=\sum_{n=-\infty}^{+\infty}X_nH_n^{(1)}(\delta_1r)e^{in\theta}+\sum_{n=-\infty}^{+\infty}Y_nH_n^{(1)}(\delta_2r)e^{in\theta} \\ \phi^f=C_1\sum_{n=-\infty}^{+\infty}X_nH_n^{(1)}(\delta_1r)e^{in\theta}+C_2\sum_{n=-\infty}^{+\infty}Y_nH_n^{(1)}(\delta_2r)e^{in\theta}\end{cases} \tag{21a}$$

$$\begin{cases}\psi^s=\sum_{n=-\infty}^{+\infty}Z_nH_n^{(1)}(\beta r)e^{in\theta} \\ \psi^f=D\sum_{n=-\infty}^{+\infty}Z_nH_n^{(1)}(\beta r)e^{in\theta}\end{cases} \tag{22a}$$

The unknown coefficients X_n, Y_n, Z_n are calculated from boundary conditions for the problem. Thus, displacements, stresses and pore pressure can be derived from the known potential functions, and then, the boundary value problem can be solved.

5. Displacements, Stresses and Pore Pressure

Using equation (4), the displacement components in the complex plane can be evaluated from:

$$\begin{cases} u_r + iu_\theta = 2\dfrac{\partial}{\partial \bar{\zeta}}\left(\phi^s - i\psi^s\right)e^{-i\theta} \\ u_r - iu_\theta = 2\dfrac{\partial}{\partial \zeta}\left(\phi^s + i\psi^s\right)e^{i\theta} \end{cases} \tag{23}$$

where u_r and u_θ are the radial and tangential components of the displacement vector respectively. To derive displacements in terms of the Hankel functions, equations (15) and (16) are used; therefore, equation (23) can be written as:

$$\begin{cases} u_r + iu_\theta = -\delta_1 \sum_{n=-\infty}^{+\infty} X_n H_{n+1}^{(1)}(\delta_1 r)e^{in\theta} \\ \qquad - \delta_2 \sum_{n=-\infty}^{+\infty} Y_n H_{n+1}^{(1)}(\delta_2 r)e^{in\theta} \\ \qquad + i\beta \sum_{n=-\infty}^{+\infty} Z_n H_{n+1}^{(1)}(\beta r)e^{in\theta} \\ u_r - iu_\theta = \delta_1 \sum_{n=-\infty}^{+\infty} X_n H_{n-1}^{(1)}(\delta_1 r)e^{in\theta} \\ \qquad + \delta_2 \sum_{n=-\infty}^{+\infty} Y_n H_{n-1}^{(1)}(\delta_2 r)e^{in\theta} \\ \qquad + i\beta \sum_{n=-\infty}^{+\infty} Z_n H_{n-1}^{(1)}(\beta r)e^{in\theta} \end{cases} \tag{24}$$

For stresses, the following relations, which are derived from the first part of equation (2), can be used:

$$\begin{cases} \sigma_r + \sigma_\theta = -2\delta^2(\lambda + G)\phi^s \\ \sigma_\theta - \sigma_r + 2i\sigma_{r\theta} = -8G\dfrac{\partial^2}{\partial \zeta^2}\left(\phi^s + i\psi^s\right)e^{2i\theta} \end{cases} \tag{25}$$

where $\sigma_r, \sigma_\theta, \sigma_{r\theta}$ are the radial, hoop and shear components of the stress tensor, respectively. Combination of the two parts of equation (25) yields:

$$\begin{cases} \sigma_r - i\sigma_{r\theta} = -\delta^2(\lambda + G)\phi^s \\ \qquad + 4G\dfrac{\partial^2}{\partial \zeta^2}\left(\phi^s + i\psi^s\right)e^{2i\theta} \\ \sigma_r + i\sigma_{r\theta} = -\delta^2(\lambda + G)\phi^s \\ \qquad + 4G\dfrac{\partial^2}{\partial \bar{\zeta}^2}\left(\phi^s - i\psi^s\right)e^{-2i\theta} \end{cases} \tag{26}$$

The pore pressure can be evaluated by using the second part of equation (2) as follows:

$$p = -\alpha Q \nabla^2 \phi^s - Q \nabla^2 \phi^f \tag{27}$$

Finally the pore pressure and stresses are written in terms of the Hankel functions, as follows:

$$\begin{cases} p = Q\delta_1^2(\alpha + C_1)\sum_{n=-\infty}^{+\infty} X_n H_n^{(1)}(\delta_1 r)e^{in\theta} \\ \qquad + Q\delta_2^2(\alpha + C_2)\sum_{n=-\infty}^{+\infty} Y_n H_n^{(1)}(\delta_2 r)e^{in\theta} \\ \sigma_r - i\sigma_{r\theta} = -\delta_1^2(\lambda + G)\sum_{n=-\infty}^{+\infty} X_n H_n^{(1)}(\delta_1 r)e^{in\theta} \\ \qquad + G\delta_1^2 \sum_{n=-\infty}^{+\infty} X_n H_{n-2}^{(1)}(\delta_1 r)e^{in\theta} \\ \qquad - \delta_2^2(\lambda + G)\sum_{n=-\infty}^{+\infty} Y_n H_n^{(1)}(\delta_2 r)e^{in\theta} \\ \qquad + G\delta_2^2 \sum_{n=-\infty}^{+\infty} Y_n H_{n-2}^{(1)}(\delta_2 r)e^{in\theta} \\ \qquad + iG\beta^2 \sum_{n=-\infty}^{+\infty} Z_n H_{n-2}^{(1)}(\beta r)e^{in\theta} \\ \sigma_r + i\sigma_{r\theta} = -\delta_1^2(\lambda + G)\sum_{n=-\infty}^{+\infty} X_n H_n^{(1)}(\delta_1 r)e^{in\theta} \\ \qquad + G\delta_1^2 \sum_{n=-\infty}^{+\infty} X_n H_{n+2}^{(1)}(\delta_1 r)e^{in\theta} \\ \qquad - \delta_2^2(\lambda + G)\sum_{n=-\infty}^{+\infty} Y_n H_n^{(1)}(\delta_2 r)e^{in\theta} \\ \qquad + G\delta_2^2 \sum_{n=-\infty}^{+\infty} Y_n H_{n+2}^{(1)}(\delta_2 r)e^{in\theta} \\ \qquad - iG\beta^2 \sum_{n=-\infty}^{+\infty} Z_n H_{n+2}^{(1)}(\beta r)e^{in\theta} \end{cases} \tag{28}$$

6. Boundary value problem

Let us consider a circular empty cavity with a radius “a”, as in the case of drained tunnel, in an infinite porous medium, which is subjected to a harmonic incident wave ϕ^{in} or ψ^{in}.

The pressure on the cavity surface is constant and may be set to zero. The stresses on this surface can be set to zero as well, therefore the following boundary conditions should be satisfied at the cavity surface:

$$\begin{cases} p = 0 \\ \sigma_r - i\sigma_{r\theta} = 0 \\ \sigma_r + i\sigma_{r\theta} = 0 \end{cases} \quad \text{at} \quad r = a \qquad (29)$$

All the wave variables are the sum of incident and scattering components; therefore the potential functions can be written as:

$$\begin{cases} \phi = \phi^{in} + \phi^{sc} \\ \psi = \psi^{in} + \psi^{sc} \end{cases} \qquad (30)$$

where ϕ^{in} and ψ^{in} are the incident, and also, ϕ^{sc} and ψ^{sc} are the scattered components of the potential functions. Thus the boundary conditions at the cavity surface are:

$$\begin{cases} p^{in} + p^{sc} = 0 \\ (\sigma_r - i\sigma_{r\theta})^{in} + (\sigma_r - i\sigma_{r\theta})^{sc} = 0 \\ (\sigma_r + i\sigma_{r\theta})^{in} + (\sigma_r + i\sigma_{r\theta})^{sc} = 0 \end{cases} \quad \text{at} \quad r = a \qquad (31)$$

Incident components of the stresses and pore pressure are derived from incident potential functions using equations (26) and (27), respectively. The scattered components of these variables are derived from equation (28).

The incident potential function is written as:

$$\Phi^{in} = \phi_0 e^{i(Kx-\omega t)} = \phi_0 e^{i(Kr\cos\theta - \omega t)} \quad \text{P1 incident wave}$$

$$\Psi^{in} = \psi_0 e^{i(Kx-\omega t)} = \psi_0 e^{i(Kr\cos\theta - \omega t)} \quad \text{SV incident wave} \qquad (32)$$

where ϕ_0 and ψ_0 are coefficients and K is the wave number. K is related to wave frequency as follows:

$$\omega = K\left[\frac{\lambda + 2G + \alpha^2 Q}{\rho}\right]^{1/2} \quad \text{P1 incident wave}$$
$$\omega = K\left(\frac{G}{\rho}\right)^{1/2} \quad \text{SV incident wave} \qquad (33)$$

Considering the following relation for P1 incident wave:

$$e^{iKr\cos\theta} = \sum_{n=-\infty}^{+\infty} i^n J_n(Kr) e^{in\theta} \qquad (34)$$

It is possible to find:

$$\begin{cases} p^{in} = \alpha Q \phi_0 K^2 \sum_{n=-\infty}^{+\infty} i^n J_n(Kr) e^{in\theta} \\ (\sigma_r - i\sigma_{r\theta})^{in} = -\phi_0 K^2 (\lambda + G) \sum_{n=-\infty}^{+\infty} i^n J_n(Kr) e^{in\theta} \\ + G\phi_0 K^2 \sum_{n=-\infty}^{+\infty} i^n J_{n-2}(Kr) e^{in\theta} \\ (\sigma_r + i\sigma_{r\theta})^{in} = -\phi_0 K^2 (\lambda + G) \sum_{n=-\infty}^{+\infty} i^n J_n(Kr) e^{in\theta} \\ + G\phi_0 K^2 \sum_{n=-\infty}^{+\infty} i^n J_{n+2}(Kr) e^{in\theta} \end{cases} \qquad (35)$$

and for SV incident wave:

$$\begin{cases} p^{in} = 0 \\ (\sigma_r - i\sigma_{r\theta})^{in} = iG\psi_0 K^2 \sum_{n=-\infty}^{+\infty} i^n J_{n-2}(Kr) e^{in\theta} \\ (\sigma_r + i\sigma_{r\theta})^{in} = -iG\psi_0 K^2 \sum_{n=-\infty}^{+\infty} i^n J_{n+2}(Kr) e^{in\theta} \end{cases} \qquad (36)$$

Finally the boundary conditions, given by equation (31), can be written as an algebraic equation:

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{Bmatrix} X_n \\ Y_n \\ Z_n \end{Bmatrix} = \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} \qquad (37)$$

where the coefficients m_{ij} are:

$$\begin{cases} m_{11} = Q\delta_1^2(\alpha + C_1)H_n^{(1)}(\delta_1 a) \\ m_{12} = Q\delta_2^2(\alpha + C_1)H_n^{(1)}(\delta_2 a) \\ m_{13} = 0 \\ \\ m_{21} = -\delta_1^2(\lambda + G)H_n^{(1)}(\delta_1 a) + G\delta_1^2 H_{n-2}^{(1)}(\delta_1 a) \\ m_{22} = -\delta_2^2(\lambda + G)H_n^{(1)}(\delta_2 a) + G\delta_2^2 H_{n-2}^{(1)}(\delta_2 a) \\ m_{23} = iG\beta^2 H_{n-2}^{(1)}(\beta a) \\ \\ m_{31} = -\delta_1^2(\lambda + G)H_n^{(1)}(\delta_1 a) + G\delta_1^2 H_{n+2}^{(1)}(\delta_1 a) \\ m_{32} = -\delta_2^2(\lambda + G)H_n^{(1)}(\delta_2 a) + G\delta_2^2 H_{n+2}^{(1)}(\delta_2 a) \\ m_{33} = -iG\beta^2 H_{n+2}^{(1)}(\beta a) \end{cases} \quad (38)$$

For the P1 incident wave, the coefficients n_i are:

$$\begin{cases} n_1 = -\phi_0 \alpha Q K^2 i^n J_n(Ka) \\ n_2 = \phi_0 K^2 i^n [(\lambda + G)J_n(Ka) - GJ_{n-2}(Ka)] \\ n_3 = \phi_0 K^2 i^n [(\lambda + G)J_n(Ka) - GJ_{n+2}(Ka)] \end{cases} \quad (39)$$

and for the SV incident wave, we have:

$$\begin{cases} n_1 = 0 \\ n_2 = -iG\psi_0 K^2 i^n J_{n-2}(Ka) \\ n_3 = iG\psi_0 K^2 i^n J_{n+2}(Ka) \end{cases} \quad (40)$$

7. Numerical Results

Figure 1 represents a cavity in an infinite isotropic elastic porous medium subjected to the harmonic wave. The cavity surface is free of stress and pressure.

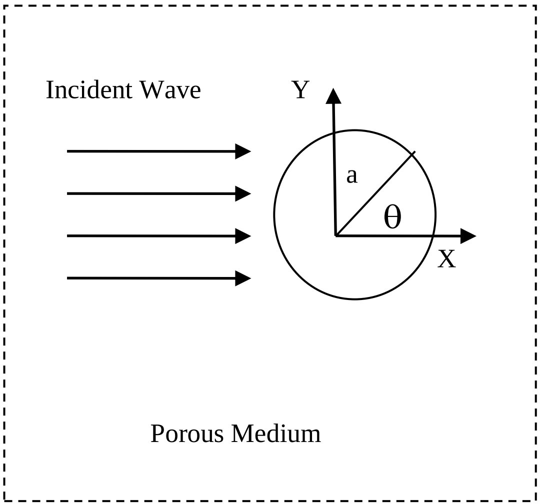

Figure 1. in an infinite porous medium subjected to the seismic wave

In all following examples, the parameters, given in Table 1, are assumed.

Table 1. Parameters used in applications

ν	n	ρ_s (kg/m^3)	ρ_f (kg/m^3)	α	Q (MPa)
0.33	0.3	2700	1000	1	66,700

A. Validation

To verify the proposed solution, a comparison between the well-known solution of solid medium and the result of this solution reduced to a solid case is made. In Figure 2, the absolute values of real and imaginary parts of dynamic stress concentration factor (D.S.C.F.) in a solid medium for the P incident wave versus various dimensionless wave numbers (Ka) and Poisson ratios are plotted ($\theta = \pi/2$). This factor is the hoop stress along $r = a$ normalized by $\phi_0(\lambda + 2G)K^2$. An excellent agreement with the results of has been observed.

In another example, the present solution is compared with the solution obtained by in a poro-elastic medium by using a boundary layer approximation. Figure 3 shows the variations of normalized pore pressure and effective stresses for the cavity in a porous medium versus the normalized radial distance (r /a) for various angel

(θ) of P1 incident wave. The stress and pore pressure values have been normalized by $\phi_0 G\beta_0^2$, in which $\beta_0^2 = \frac{\rho\omega^2}{G}$.

In this example, the following values are chosen: $G = 2000$ MPa, $k = 10^{-8}\ m^3 s/kg$ and $Ka = 1$. As it can be observed, there is a very good agreement between these results and those from the solution of the problem.

B. Parametric study

To investigate the effects of different parameters, a parametric study is performed for the same geometry and with the mentioned parameters. In this study the cavity is subject to the P1 incident wave.

Figure 4 gives the ratio of effective hoop stress to the parameter K^2 versus the dimensionless wave number, Ka ($r = a$, $\theta = \pi/2$) for various soil shear modules. It can be observed that the hoop stress increases with increasing shear modulus. Figure 5 represents the ratio of effective hoop stress to the parameter K^2 versus dimensionless wave number, Ka ($r = a$, $\theta = \pi/2$) for different permeability coefficient, k (m/s). Permeability does not have a significant effect on the stress response. Figure 6 shows the ratio of effective hoop stress to the parameter K^2 for various points at the edge of the cavity and various values of shear modulus of the soil ($Ka = 1$, $r = a$). Figure 7 gives the ratio of effective radial stress to the parameter K^2 for various points at the edge of the cavity and various values of shear modulus of the soil ($Ka = 1$, $r = 2a$). As it can be seen, the value of stresses increases with increasing shear modulus. Figure 8 represents the ratio of pore pressure to the parameter K^2 versus the dimensionless wave number, Ka ($r = 2a$, $\theta = \pi/2$) for different permeability coefficients. The same effect on the pore pressure response is observed. The contribution of the fast, slow and shear waves on the scattered radial displacement (u_r^{sc}) response is presented in Figure 9.

This Figure gives the displacement amplitude of the fast, slow and shear components of scattered waves normalized by the incident wave displacement ($Ka = 1, \theta = \pi$). It can be found that the most significant contribution of displacement is due to the fast wave. The shear wave has a medium contribution. The contribution of slow wave is very small.

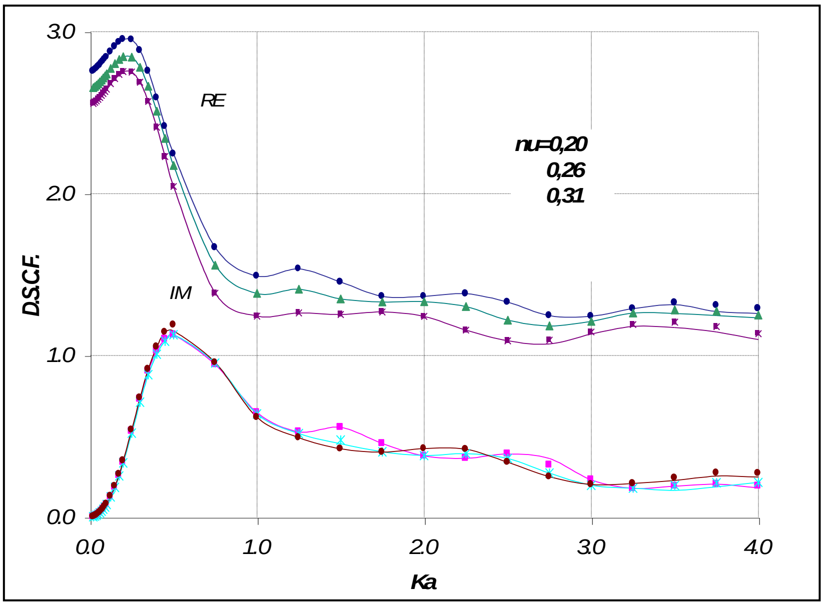

Figure 2. Absolute values of real and imaginary parts of D.S.C.F versus dimensionless wave number for various Poisson ratio in a solid medium ($\theta = \pi/2$) .Solid line: present study, point line

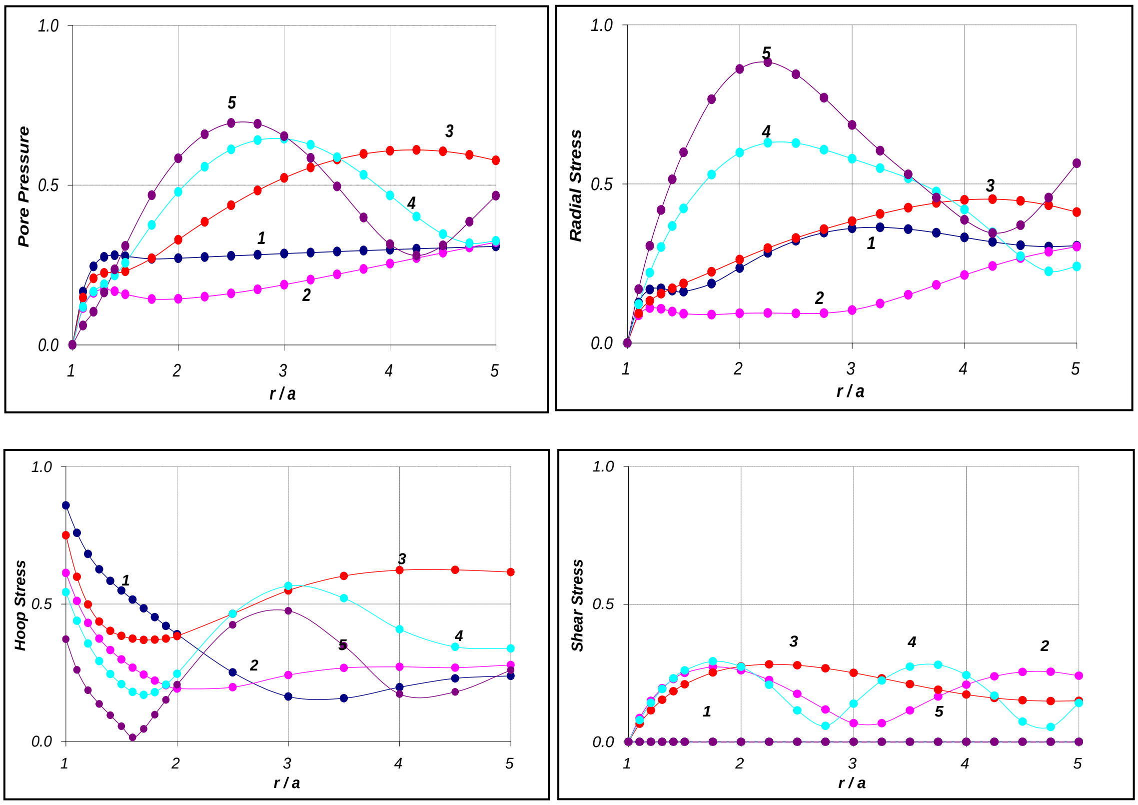

Figure 3. Values of normalized pore pressure and effective stresses versus distance from the cavity due to P1 incident wave.
(1: $\theta = 0$, 2: $\theta = \pi/4$, 3: $\theta = \pi/2$, 4: $\theta = 3\pi/4$, 5: $\theta = \pi$).Solid line: present study, point line

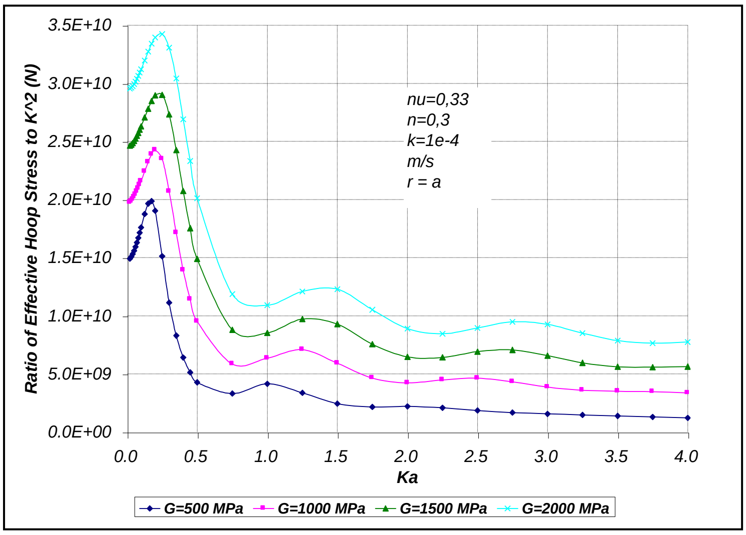

Figure 4. Ratio of effective hoop stress to K^2 versus dimensionless wave number for various shear modulus of the soil ($\theta = \pi/2$)

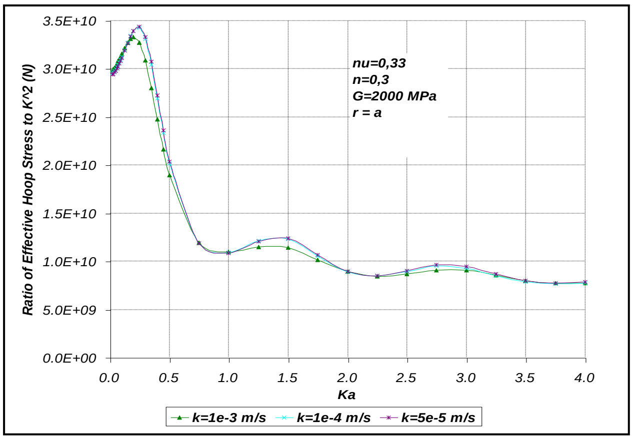

Figure 5.Ratio of effective hoop stress to K^2 versus dimensionless wave number for various permeability parameters of the soil ($\theta = \pi/2$)

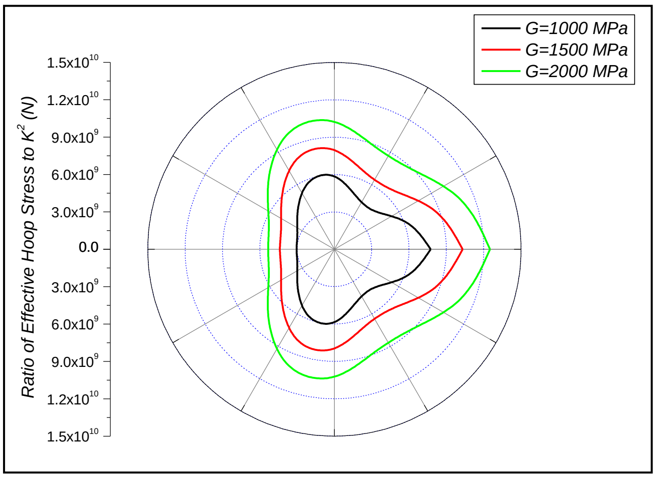

Figure 6. Ratio of effective hoop stress to K^2 for various shear modulus of the soil ($k = 10^{-4}$ m/s , $Ka = 1$, $r = a$)

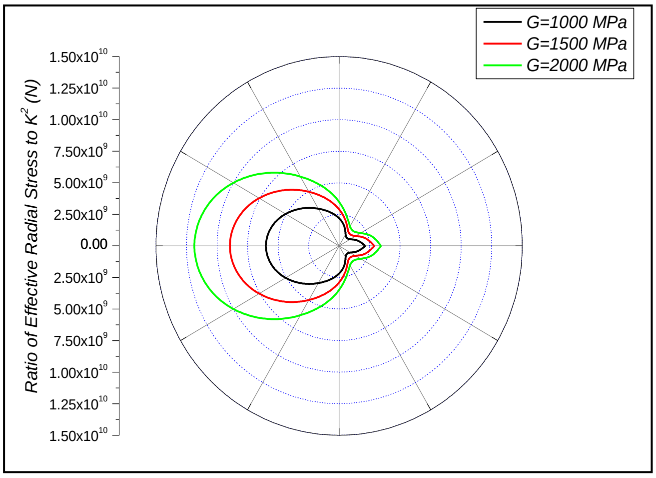

Figure 7. Ratio of effective radial stress to K^2 for various shear modulus of the soil ($k = 10^{-4}$ m/s , $Ka = 1$, $r = 2a$)

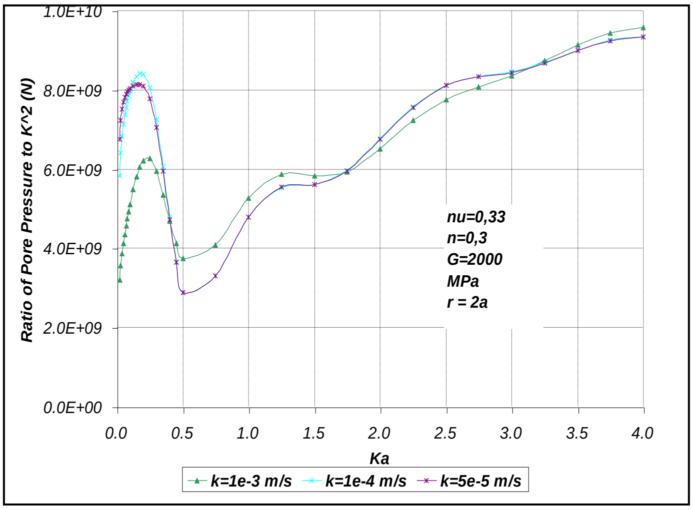

Figure 8. Ratio of pore pressure to K^2 versus dimensionless wave number for various permeability parameters of the soil ($\theta = \pi/2$)

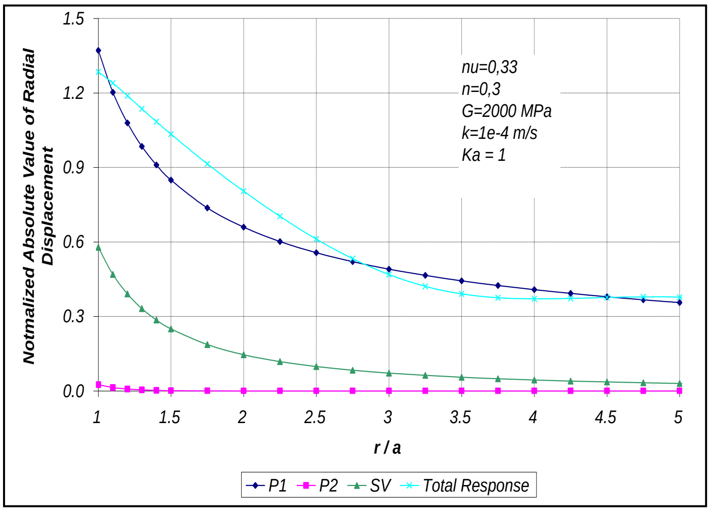

Figure 9. Normalized amplitude of scattered radial displacement versus radial distance from cavity ($\theta = \pi$)

7. Conclusion

This paper presents an analytical solution for the scattering of harmonic waves by a cavity in an infinite isotropic porous medium based on Biot wave propagation theory. The presented formulation is for a 2D plane strain case. This analytical solution is obtained in series of Hankel Functions by applying complex functions to the governing equations. The validation of proposed solution is shown by various numerical examples. A parametric study including the effects of fluid compressibility changes, shear modulus and permeability variations, several wave numbers and wave types (fast, slow and shear waves) are performed. One can note that the effect of shear modulus changes on the induced pore pressure and effective stress variations is more significant than that of permeability.

The contributions of the fast wave and shear wave in the radial displacement are more important than that of slow wave. Since in this method the radiation condition is satisfied directly in a simple manner, this analytical solution seems to be a powerful and efficient way for evaluating the wave propagation and scattering in unbounded isotropic poro-elastic saturated domains.

8. References

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